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Free space dynamics of Laguerre-Gaussian-vortex beam

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Abstract

In this paper, we analytically demonstrate the propagation of a useful structured light laser beam; namely, the vortex Laguerre-Gaussian beam, the latter is an eigenmode of free space propagation, which is invariant under propagation in lossless systems. Through some numerical simulations, we show the main spatial features of such interesting beams.

Keywords : Optical vortex, vortex Laguerre-Gaussian beams, propagation

1. Introduction

In recent years, research interests have been focused on the study and generation of light beam vortices [1-2]. Vortices are a common phenomenon that is widely found in different physics areas including optics [3-4]. While, an optical vortex is an optical field with zero intensity and an undefined phase value, due to the twisting of the light field around the propagation axis [5]. Such light is characterized by a helical wavefront with a phase singularity [6]. In optics, phase singularity is a phase defect point (in 2D) or a line (in 3D) where the phase is indeterminate and the amplitude equals zero.

The most common optical vortices are vortex Laguerre-Gaussian beams, these latter have concentric rings and a spiral wavefront described by $exp(il\varphi)$, where (l) is a topological charge [7]. Due to their distinct characteristics, these beams have found many applications, in particular; telecommunications [8], spatial multiplexing [9], and optical tweezers [10].

In this work, we discuss the propagation of different vortex Laguerre-Gaussian beams in the longitudinal and transversal planes, showing the intensity and the phase of different obtained shapes for different topological charges.

2. Free-space dynamics of Laguerre Gaussian vortex beam:

The optical filed of Laguerre-Gaussian beam at $z = 0$ in cylindrical coordinates is expressed as:

$$E_{LG}(r_0,\theta,0) = E_0\left(\sqrt{2}\frac{r_0}{w_0}\right)^{|l|} L_p^{|l|}\left(2\frac{r_0^2}{w_0^2}\right) e^{\left(-\frac{r_0^2}{w_0^2}\right)} e^{-il} \quad . \tag{1}$$

Where r_0, θ represent the radial and azimuthal coordinates, respectively. $L_p^{|l|}$ denotes the Laguerre polynomial with p and l indices.
The dynamics propagation of vortex laser beams through ABCD system is governed by the Collins integral given in cylindrical coordinates by [11]

$$E_{out}(r,\varphi,z) = \frac{ik}{2\pi B}.e^{(ikz)} e^{\left(\frac{ikD}{2B}r^2\right)} \int_0^{2\pi}\int_0^{\infty} E_{LG}(r_0,\theta,0) \mathrm{Exp}\left(\frac{ik}{2B}[A{r_0}^2 - 2rr_0\cos(\theta-\varphi)]\right) r_0 dr_0 d\theta \tag{2}$$

For free space propagation the Matrix elements A=1 and B=z, the Collins integral becomes:

$$E_{out}(r,\varphi,z) = \frac{ik}{2\pi z}.e^{(ikz)} e^{\left(\frac{ik}{2z}r^2\right)} \int_0^{2\pi}\int_0^{\infty} E_{LG}(r_0,\theta,0) \mathrm{Exp}\left(\frac{ik}{2z}[{r_0}^2 - 2rr_0\cos(\theta-\varphi)]\right) r_0 dr_0 d\theta \tag{3}$$

When we substitute equation (1) into equation (3), we obtain:

$$E_{out}(r,\varphi,z) = \frac{ik}{2\pi z}.e^{(ikz)} e^{\left(\frac{ik}{2z}r^2\right)} \int_0^{2\pi}\int_0^{\infty} E_0\left(\sqrt{2}\frac{r_0}{w_0}\right)^{|l|} L_p^{|l|}\left(2\frac{r_0^2}{w_0^2}\right) e^{\left(-\frac{r_0^2}{w_0^2}\right)} e^{-il\theta}\, Exp\left(\frac{ik}{2z}[{r_0}^2 - 2rr_0\cos(\theta - \varphi)]\right) r_0 dr_0 d\theta \tag{4}$$

We use the following integral formulas [12]:

$$\int_0^{2\pi} \mathrm{Exp}\left(-\frac{ik}{B}[r_1 r_2\cos(\varphi_1-\varphi_2)]\right) e^{-il\varphi_1} d\varphi_1 = 2\pi(-i)^l \exp(-il\varphi_2) J_l\left(\frac{kr_1r_2}{B}\right) \tag{5}$$

With J_l is the Bessel function of order l.
And

$$\int_0^{\infty} x^{l+1}\mathrm{Exp}(-\beta x^2) L_p^l(\alpha x^2) J_1(yx)\,dx = 2^{-l-1}\beta^{-l-p-1} \times (\beta-\alpha)^p y^l \mathrm{Exp}\left(-\frac{y^2}{4\beta}\right) L_p^l\left(\frac{\alpha y^2}{4\beta(2-\beta)}\right) \tag{6}$$

After some algebra, the propagation of the LG beam is expressed as:

$$E_{out}(r,\varphi,z) = \frac{ik}{2w_0^2 z}.e^{(ikz)}.e^{\left(\frac{ik}{2z}r^2\right)}.\left(-i\frac{\sqrt{2}kr}{2zw_0^2}\right)^{|l|}\left(\frac{1}{w_0^2}-\frac{ik}{2z}\right)^{-l-p-1}$$
$$\times \exp(-il\varphi)\left(\left(\frac{1}{w_0^2}-\frac{ik}{2z}\right)-2\right)^p \mathrm{Exp}\left(-\frac{k^2r^2}{4z^2\left(\frac{1}{w_0^2}-\frac{ik}{2z}\right)}\right) L_p^{|l|}\left(\frac{k^2r^2}{2z^2\left(\frac{1}{w_0^2}-\frac{ik}{2z}\right)\left(2-\left(\frac{1}{w_0^2}-\frac{ik}{2z}\right)\right)}\right) \tag{7}$$

By multiplying the filed expression by its conjugate, the intensity becomes:

$$I_{out} = E_{out}(r,\varphi,z) \times E_{out}^*(r,\varphi,z) \tag{8}$$

3. Results:

In this section, in order to investigate the propagation behavior of vortex Laguerre-Gaussian beam, we use equations (7), (8), where we present the intensity distribution of the beam as well as its phase.

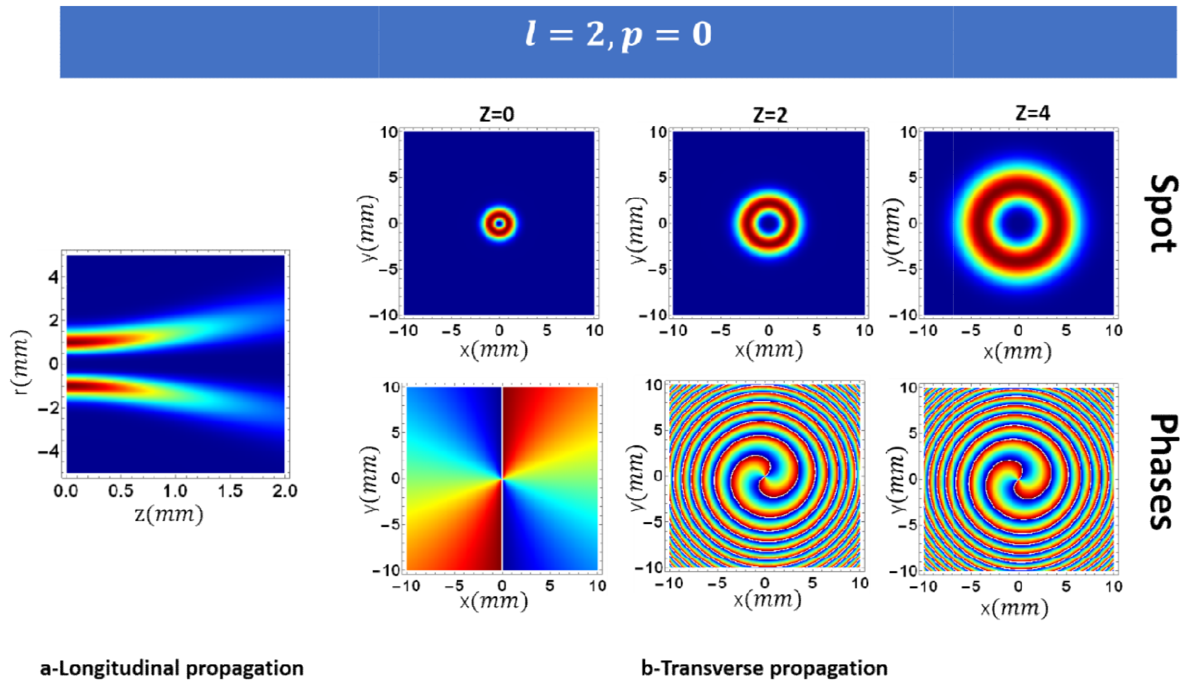

Figure 1: Longitudinal and transverse propagation of LG_0^2 beams representions in both intensity and phase.

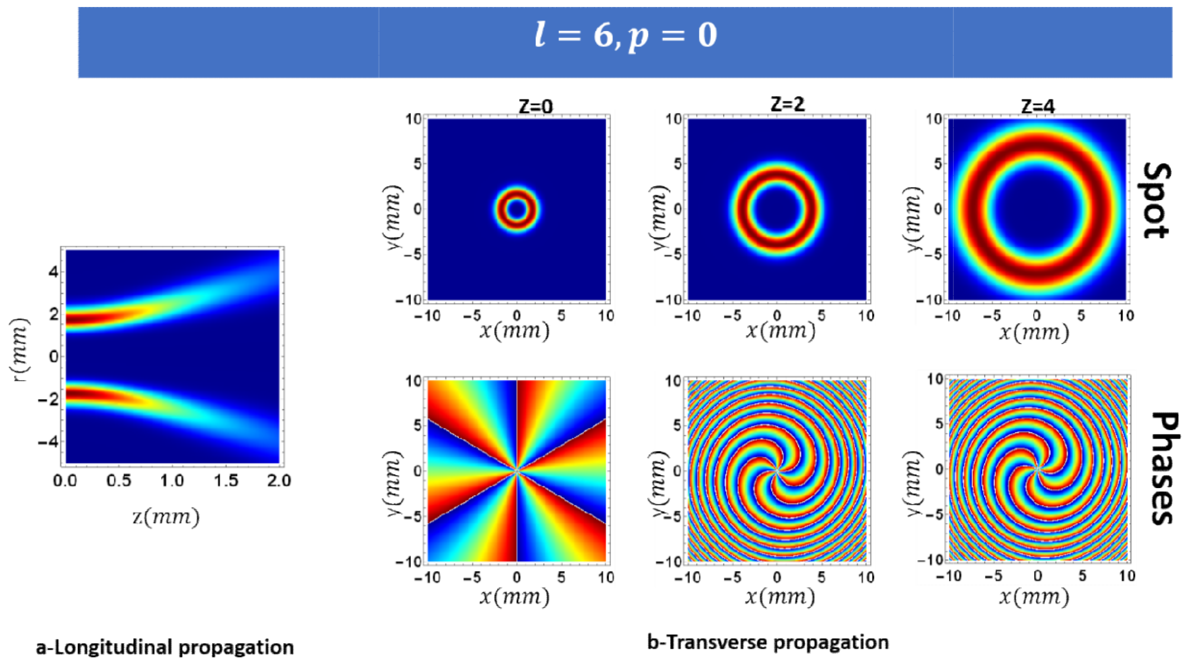

Figure 2: Longitudinal and transverse propagation of LG_0^6 beams representions in both intensity and phase.

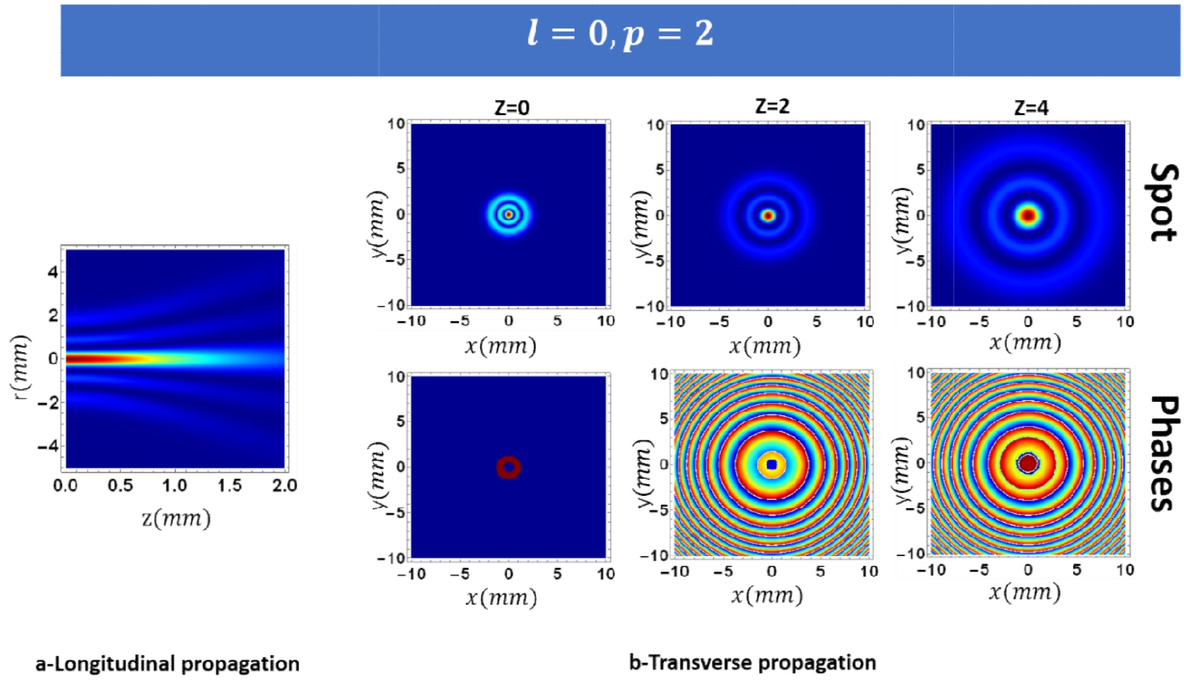

Figure 3: Longitudinal and transverse propagation of LG_2^0 beams representions in both intensity and phase.

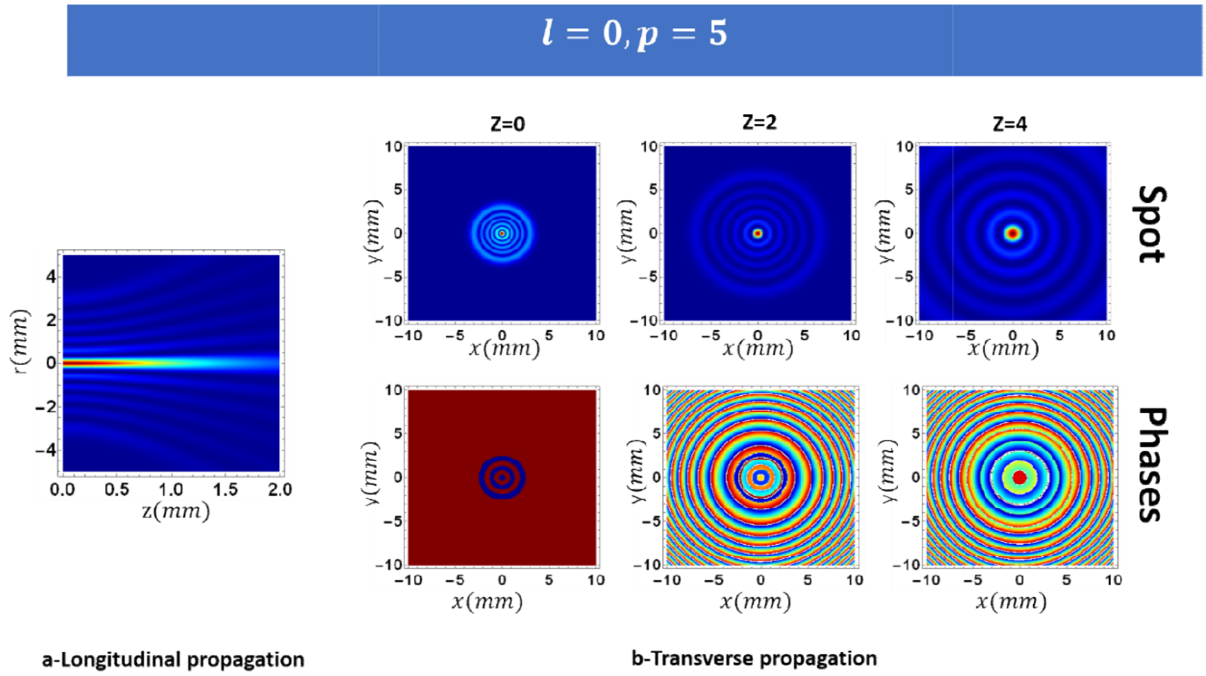

Figure 4: Longitudinal and transverse propagation of LG_5^0 beams representions in both intensity and phase.

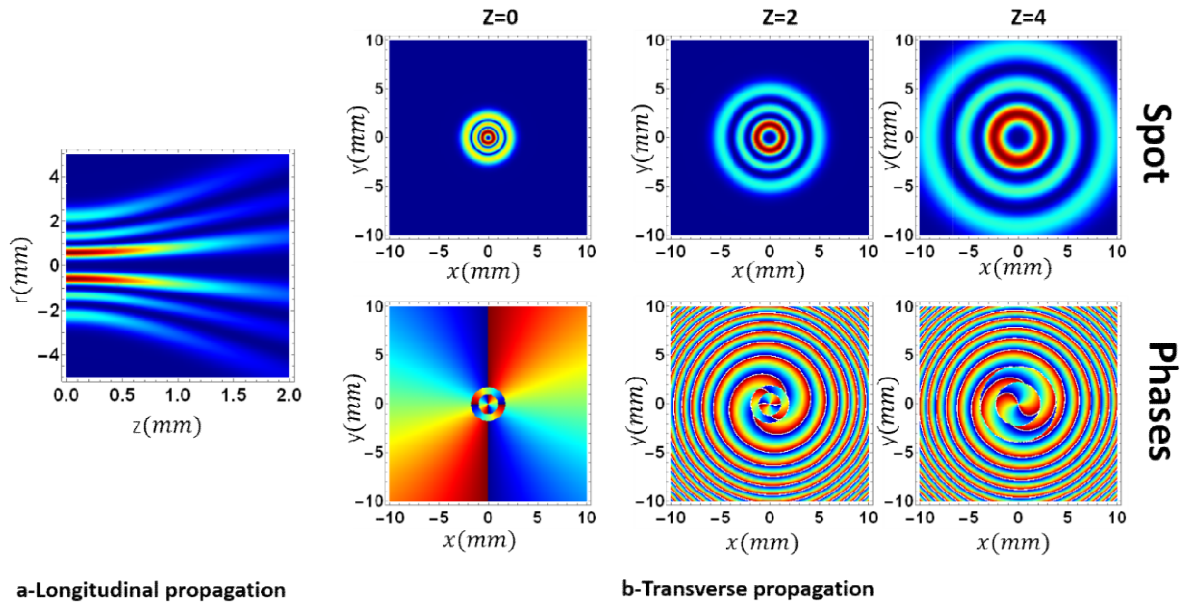

Figure 5: Longitudinal and transverse propagation of LG_2^2 beams represention in both intensity and phase.

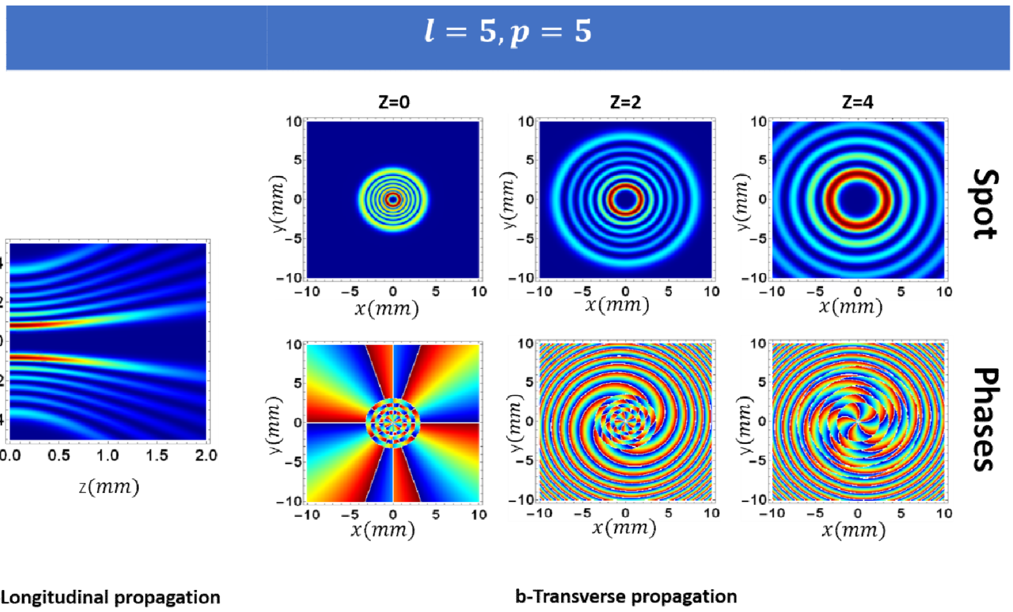

Figure 6: Longitudinal and transverse propagation of LG_5^5 beams represention in both intensity and phase.

4. Discussion:

From the obtained results, the vortex Laguerre-Gaussian beam keeps its shape during the propagation. In figures, (1,2,5,6) the beam has a dark center with large ring, which depends on the beam order. In addition, the phase rotates clockwise during propagation. Meanwhile in figures, (3,4) the vortex Laguerre-Gaussian beam has a bright center surrounded by concentric rings in both intensity and phase profiles. The number of the bright rings depends on the radial order.

5. Conclusion:

In conclusion, we have demonstrated analytically using Collins integral and ABCD matrix, that the Laguerre-Gaussian beam remains vortex Laguerre-Gaussian beam in free space propagation, which means (the beam keeps its structure during propagation)
We have confirmed the obtained results by simulations, such us the invariance structure of the amplitude and phase under free space propagation. Such conclusion, make these beams a good candidate for free space communication, spatial multiplexing and metrology.

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