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Non-Gaussianity for Harmonic Oscillator isospectral potentials

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Abstract

We address non-Gaussianity of quantum states of Harmonic Oscillator isospectral potentials introduced in the super-symmetric quantum mechanics (SUSYQM) approach. We show explicitly that the ground and thermal states are non-Gaussian and the non-Gaussianity feature is independent of the energy spectrum of the considered potential.e abstract should consist of a single paragraph containing no more than 300 words. It should be a summary of the paper and not an introduction. Your abstract should give readers a brief summary of your article. It should concisely describe the contents of your article.

Keywords: isospectral potentials; non-Gaussianity; thermal states; Harmonic oscillator.

1. Introduction

Within the SUSYQM formalism, the non-uniqueness of the factorization has been exploited to generate one- parameter family of non-linear potentials which are non-singular, exactly solvable, and strictly isospectral to the shifted harmonic oscillator potential (SHO) [1]. On the other hand, the generation, manipulation, and detection of non-Gaussian states have aroused growing interest in quantum optics, and quantum information [2, 3]. These quantum states are widely used in several protocols in quantum communication [4].

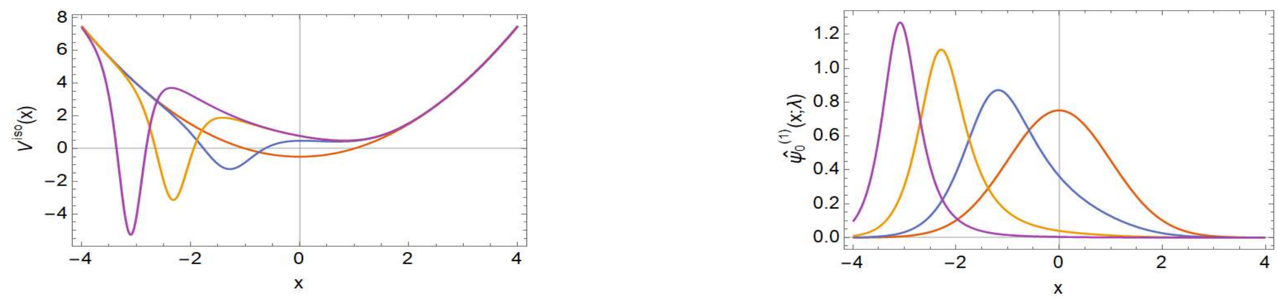
2. Methodology

Using SUSYQM techniques [1]and particularly the Ricatti $V^{(1)}(x)$ equation, we analyze the SHO potential and further find its isospectral potentials:

$$\hat{V}^{(1)}(\lambda;x)=\frac{1}{2}\left(\frac{16\lambda^2 e^{-2x^2}}{\pi\left(\sqrt{2}\lambda(\mathrm{erf}(x)+1)+2\right)^2}+\frac{8\sqrt{\frac{2}{\pi}}\lambda e^{-x^2}x}{\sqrt{2}\lambda(\mathrm{erf}(x)+1)+2}+x^2-1\right) \tag{1}$$

Which has no singularities when $\lambda>1/\sqrt{2}$and the limit $\lambda\to 0$ corresponds to the SHO potential $V^{(1)}(x)$. Its eigenvalues are , that $\hat{E}_n^{(1)}=n$ is, identical to those of the SHO. The normalized ground state wave functions are given by:

$$\hat{\psi}_0^{(1)}(\lambda;x)=\frac{2\sqrt{\lambda\left(\lambda\left(\sqrt{2}\lambda+5\right)+4\sqrt{2}\right)+2}\,e^{-\frac{x^2}{2}}}{\sqrt[4]{\pi}\sqrt{\left(\lambda+\sqrt{2}\right)^2}\left(\sqrt{2}\lambda(\mathrm{erf}(x)+1)+2\right)} \tag{2}$$



In the left panel we show $\hat{V}^{(1)}(\lambda;x)$ for λ=0(blue),λ=10(orange),λ=10^3(green),λ=10^5(red).
In the right panel we show the corresponding ground state wavefunctions, $\hat{\psi}_0^{(1)}(\lambda;x)$

Fig1.

Analyzing ground and thermal states (states at thermal equilibrium) of some harmonic oscillator isospectral potentials (Figure 1), we evaluate the corresponding non-Gaussianity based on the quantum relative entropy (QRE) measure $\delta[\rho]$ between the state under examination $\rho$and a reference ρ_G Gaussian state [2, 3, 4].

$$\delta[\rho] = h\left(\sqrt{\det\sigma}\right) + Tr\left(\rho\log\rho\right) \tag{3}$$

where $\sigma$is the covariance matrix and $h(x)$ is a function given by:

$$h(t) = (t+\frac{1}{2})\ln(t+\frac{1}{2}) + (t-\frac{1}{2})\ln(t-\frac{1}{2}) \tag{4}$$

3. Results and discussion

As is apparent from the plot below (Figure 2), the two measures are both monotone with respect Riccati (deformation) parameter λ , as long as the value of λ is not too large. For increasing λ, the measure relevant to thermal state continues to grow whereas the associated measure of ground state has a maximum.

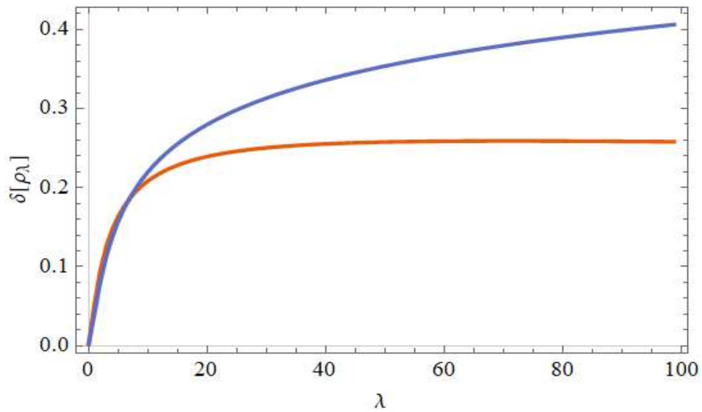


Fig.2. QRE based nonG δ[ρ] as a function of the parameter λ. we considered the isospectral SHO quantum states: ground state (red line) and thermal state (blue line).

4. Conclusion

Addressing the explicit example of Harmonic oscillator isospectral potentials we have been able to find that non-Gaussianity based on the quantum relative entropy (QRE) measure in such continuous variable systems, is in monotonic relation with the Riccati (nonlinearity) parameter λ, Although our conclusions about the non-linear features of this kind of potentials have been gathered by looking at the non-Gaussian properties of both the ground states and the thermal states which account for the whole spectrum.

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